

**SUMMARY OF RESEARCH REPORT**

**A COLLABORATIVE NASA AND CARNEGIE MELLON PROGRAM TO STUDY**

**THE APPLICATION OF PROBABILISTIC METHODS  
TO THE MISTUNING PROBLEM**

**AWARD NUMBER: NCC3 – 1058**

**START DATE: 6/1/2003**

**BY**

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**DECEMBER 2004**

## ABSTRACT

FMM is a reduced order model for efficiently calculating the forced response of a mistuned bladed disk. FMM ID is a companion program which determines the mistuning in a particular rotor. Together, these methods provide a way to acquire data on the mistuning in a population of bladed disks, and then simulate the forced response of the fleet. This process is tested experimentally, and the simulated results are compared with laboratory measurements of a "fleet" of test rotors. The method is shown to work quite well. It is found that accuracy of the results depends on two factors: the quality of the statistical model used to characterize mistuning, and how sensitive the system is to errors in the statistical modeling.

## 1. INTRODUCTION

This report discusses the work performed under the NASA grant Award Number NCC3 - 1058. It focuses on applying probabilistic methods to the bladed disk mistuning problem. As described in the Annual Progress Report, it was found that the advanced techniques used in the NASA probabilistic computer code, NESSUS, were not applicable to the turbine blade mistuning problem because the response functions had discontinuous slopes. As a result, all of the probabilistic calculations discussed in this report were computed using Monte Carlo simulations. This report focuses on an experimental and analytical study on the use of probabilistic methods that was conducted at Carnegie Mellon as part of the NASA funded effort. This section describes the mistuning problem and the method of representing mistuning that will be used in the study.

Bladed disks are generally designed to be cyclically symmetric. However, in practice manufacturing effects, non-uniform material properties, and wear cause each blade to be slightly different from the rest. These blade-to-blade variations are known as mistuning. The resonant amplitudes of turbine blades are very sensitive to these small variations in the blade properties. Therefore, mistuning can significantly amplify the vibratory response of some blades, and cause them to fail from high cycle fatigue. Srinivasan provides a thorough review of the topic [1].

One area of mistuning research has focused on the development reduced order models to efficiently predict the forced response of a mistuned bladed disk. A variety of reduced order models have been developed by researchers at Carnegie Mellon University [2-3], the University of Michigan [4], and Imperial College [5]. Although these methods have been shown to agree extremely well with finite element simulations of a full mistuned rotor, some have had difficulty predicting the response of actual hardware [6]. These results suggest that the source of the error may lie in our inability to determine the correct input parameters to the ROMs.

Therefore, researchers subsequently developed methods to accurately measure the mistuning in a bladed disk. The first advanced mistuning identification method was created by Judge and Pierre [7]. Their technique uses measurements of the bladed disk system as a whole to infer the mistuning of individual blades. More recently, Kim and Griffin developed a similar technique which is applicable to veering regions and high frequency modes [8]. However, the techniques of Judge and Kim require a finite element model of the system, and significant analysis to identify the mistuning in an IBR. A much simpler approach is the FMM ID method developed by Feiner and Griffin [9,10].

FMM ID is based on the Fundamental Mistuning Model (FMM), a simple reduced order model for mistuned bladed disks [3]. Like the methods of Judge or Kim, FMM ID also uses measurements of the whole

assembly to infer blade frequencies, but it is completely experimental. FMM ID does not require a finite element model. This makes FMM ID easier to use than the other methods. Furthermore, we have found that in practice it is extremely difficult to obtain a finite element model that accurately reflects a component's true geometry and boundary conditions. Such inaccuracies may lead to errors when using the methods of Judge and Kim. However, since FMM ID relies only on experimental data, it is not hindered by the quality of a finite element model.

FMM ID can also experimentally determine the natural frequencies that a bladed disk would have if it were tuned. The combination of tuned frequencies and mistuning provide enough information for FMM to predict the forced response of a bladed disk based solely on experimentally measured data.

Both FMM and FMM ID have been verified experimentally in deterministic calculations [10]. However, it has also been proposed that these methods may be used for probabilistic analysis. The idea is to use FMM ID to acquire data on the mistuning and tuned system frequencies in a population of bladed disks. Then, perform Monte Carlo simulations with FMM to assess the response of the fleet. In this paper, we test this process experimentally, and then compare the simulated results with measurements of a "fleet" of test rotors.

This paper is organized as follows. In Section 2, we summarize the FMM and FMM ID methods. Then, Section 3 describes the benchmark experiments. In Section 4 we discuss the probabilistic analysis, and compare our simulation results with experimental data. Finally, the key results are summarized in Conclusions.

## 2. FMM AND FMM ID

### 2.1 FMM

FMM is a simple reduced order model of mistuned bladed disk vibration. The method is a simplification of the Subset of Nominal Modes theory (SNM) developed by Yang and Griffin [2] and is designed for use in low frequency modes such as first bending and first torsion. One of the advantages of this approach is that it reduces the mistuning problem to its most basic elements. As a result, FMM requires a minimum number of input parameters, and it's extremely easy to use. This large simplification also makes FMM extremely efficient. When performing Monte Carlo simulations of forced response, FMM can simulate the response of about 200 disks per second on a 2 GHz PC.

The FMM method only requires two sets of input parameters to calculate the effect of mistuning on the mode shapes and frequencies of a bladed disk. Consider the eigenvalue problem solved by FMM to calculate mistuned modes and frequencies,

$$(\Omega^2 + \hat{A})\tilde{\beta}_j = \omega_j^2 \tilde{\beta}_j \quad (1)$$

Notice that the equation has only two input matrices,  $\Omega^2$  and  $\hat{A}$ .  $\Omega^2$  is a diagonal matrix of the tuned system frequencies squared. This term describes the nominal system. The matrix  $\hat{A}$  characterizes the mistuning.  $\hat{A}$  is composed of the blade frequency deviations, which are defined as the difference in each blade's frequency from the average value. Thus, FMM shows that the effect that mistuning has on a system is

completely defined by only two sets of parameters: the tuned system frequencies, and the blade frequency deviations.

This result has a large implication to probabilistic analysis because it minimizes the number of random variables which must be accounted for when calculating the response of mistuned systems. Since mistuning may be characterized by the frequencies of the blades, it is not necessary to separately model the variations in blade geometry and material properties. All we need to account for is the net effect of these variations on the blade frequencies.

Therefore, it is important to be able to accurately measure the frequencies of individual blades. Blade frequencies are often difficult to measure directly, particularly in the case of IBRs, where the blades cannot be removed for individual testing. But FMM provides a method for blade frequency identification: FMM ID.

## **2.2 FMM ID**

Recall that the FMM eigenvalue problem, Eq. (1), is used to calculate the effect of mistuning on the mode shapes and natural frequencies of a bladed disk. The equation takes as input, information on the nominal system as well as the way it's mistuned. With this data, the expression can be solved for the mode shapes and natural frequencies of the mistuned bladed disk. However, we could alternatively solve the problem in reverse. Suppose we measured the modes and natural frequencies of a mistuned rotor. We could then formulate an inverse problem to Eq. (1) [9]. The solution to this inverse problem is the mistuning of each blade, as well as the natural frequencies the system would have if it were tuned. This is the basis of FMM ID, and it is shown schematically in Fig. 1.

In practice, the mistuned system modes and frequencies are measured through standard modal testing techniques. This involves measuring a set of transfer functions, and then extracting modes with modal curve fitting software. For the purpose of FMM ID, the modes only need to be measured at one point per blade.

FMM ID does not require any finite element data. Thus, it provides us with a way to determine all of the key mistuning parameters experimentally.

## **2.3 Probabilistic Application of FMM and FMM ID**

By measuring multiple bladed disks of the same design, FMM ID can collect data on mistuning and tuned system frequencies, which can later be used for probabilistic analysis.

Once the data is collected, we can construct statistical models of the mistuning and tuned system frequencies. Then, we can use those statistical models with FMM to perform Monte Carlo simulations of the fleet.

The goals of this research are to apply this technique to an academic rotor, and compare our simulations with experimental data. This will allow us to explore some of the challenges of probabilistic mistuning analysis.

## **3. BENCHMARK EXPERIMENT**

The experimental approach is to first generate benchmark data with which to compare our Monte Carlo simulation results. This requires measuring the forced response of multiple bladed disks of the same design. Next, we can apply the probabilistic procedure discussed in the previous section to simulate the response of a fleet of similarly constructed disks.

The first step in performing the benchmark experiment was to obtain a tuned bladed disk, which we could later mistune in a controlled fashion. Figure 2 shows the academic IBR used for experiment. To tune the structure, we first used FMM ID to measure the frequency of every blade. Since the blades on this disk have a simple beam-like geometry, we were then able to use beam theory to calculate the appropriate length change for every blade to compensate for its mistuning. Finally, we trimmed the blade lengths accordingly. Figure 3 shows the frequency response function (FRF) of the rotor before and after tuning. Notice that prior to tuning, the structure's mistuning caused the repeated natural frequencies to split, producing additional peaks of the FRF. After tuning, the splitting was eliminated in most of the modes. Thus, the disk was successfully tuned.

Then, the disk was mounted in the test fixture shown in Fig. 4. The disk was mistuned by adding masses to the blade tips. The masses were selected to produce variations in the blade frequencies that were approximately normally distributed with a standard deviation equal to 2% of the nominal blade frequency. Note that a mean shift in the mistuning is mathematically equivalent to a mean shift in the tuned system frequencies. Therefore, we defined the mean mistuning to be zero, and measured the corresponding tuned system frequencies through FMM ID.

We excited the disk with an array of electro-magnets positioned under the blade tips, Fig. 4. The magnets produced an engine style excitation, while the disk remained stationary. The engine style excitation system at Carnegie Mellon is similar to the one developed by Jones and Cross at the Air Force Research Laboratories [11]. The bladed disk was excited over an appropriate frequency range to simulate the effect of an engine order crossing with the first bending modes. The vibratory response of the blades was measured at each blade tip by using a scanning laser vibrometer, Fig 5a. Laser vibrometers are ideal tools for mistuning measurements since they are very accurate, non-contacting sensors which don't alter the mistuning of the structure.

This measurement process was repeated with 10 different mistuning patterns, each drawn from the same normal distribution. This effectively gave us measurements of 10 different disks from the same population. On each test, we recorded the peak amplitude of every blade over the frequency range of interest, Fig 5b. Since each disk has 24 blades, this produced a total of 240 peak amplitude measurements, which we will later use for comparison with our simulation results. Furthermore, every "disk" was tested with four different engine order excitations: 1E, 3E, 6E, and 9E. Thus, we will be able to assess the accuracy of the probabilistic analysis method over a wide variety of excitation conditions.

#### **4. PROBABILISTIC ANALYSIS**

Next, we followed the probabilistic analysis process outlined in Section 2.3. We proceeded as if we knew nothing about the way the disks were mistuned.

##### **4.1 Single Disk Model**

We performed a modal analysis on one of the 10 test “disks.” Then, the measured modes were used in FMM ID to determine the structure’s mistuning and tuned system frequencies, Fig 6. This represents the crudest possible data for forming a statistical model of the bladed disk parameters. In practice, it is advisable to identify the parameters of multiple bladed disks to form a reliable model of the random variables. However, for the purpose of this study, we would like to assess the effect of a crude statistical model on the accuracy of the subsequent Monte Carlo simulations. Therefore we formed an approximate statistical model based on this limited set of data. With only one measure of the tuned system frequencies, we have no basis on which to model variability. Therefore, the tuned frequencies were treated as fixed. The blade frequency deviations, however, were modeled as a random variable. Figure 7 shows a normal plot of the 24 blade frequency deviations from this disk. Notice that the data approximately falls on a straight line. This indicates that data is roughly normal. Therefore, the mistuning was modeled as being normally distributed with a mean and standard deviation given by the sample values of 0 and 1.52% respectively. However, it must be noted that there is substantial uncertainty in these parameters. For instance, the 95% confidence interval on the standard deviation covers a range from 1.18% to 2.13%. That’s nearly a factor of 2 uncertainty in the model parameter.

Based on this rough model, we performed Monte Carlo simulations of the bladed disk population using FMM. These simulations were repeated for all four engine orders measured in the benchmark experiments. In each case, we simulated the forced response of 1000 bladed disks. The results are shown in Fig. 8. Each plot contains the CDF of all 240 peak blade amplitudes from the experiment, and a corresponding CDF constructed from the simulation results. For clarity, the plots are shown on a normal probability scale. The agreement is surprisingly good considering that the experimental CDFs only contain 240 data points, and are likely not converged in the tails. Furthermore, the statistical model used in the simulations was inaccurate. This suggests that the response is relatively insensitive to errors in the statistical model. To better understand this behavior, we performed a sensitivity analysis.

## 4.2 Sensitivity Analysis

A small change in the standard deviation used in our statistical models will produce a shift in the simulated CDF, Fig. 9. In general, the shift will not be uniform over the full range of the CDF, as shown in the figure. Thus, one method for measuring sensitivity is to plot the change in the CDF due to a perturbation in the standard deviation. This analysis was performed about a nominal standard deviation of 2%, and was repeated for all four engine orders, Fig. 10. Notice that the 6E and 9E cases are nearly zero across the full range of probability. Thus, this analysis suggests that this disk’s response to 6E and 9E excitations is very insensitive to errors in the standard deviation. This is consistent with the CDFs of Fig. 8. In particular, consider the 9E CDF. Notice that despite a large error in the statistical model used to generate the simulated curve, it agrees extremely well with the experimental data from about 10% to 90% cumulative probability. The discrepancy seen in the tails is most likely a result of insufficient experimental data to produce a converged experimental CDF in those regions.

Next, consider the 1E case. The sensitivity plot indicates that the system’s response to a 1E excitation is relatively sensitive to errors in the statistical model for most of the probability range. However, in the vicinity of 85% cumulative probability, the 1E line passes through zero on the sensitivity plot, and is therefore much less sensitive. Again, this is consistent with the CDF plot for this case, which shows good agreement between the simulation and experiment around 85%, yet larger discrepancies away from that area. This suggests that

much of the error seen in the low engine order simulations is due to a poor statistical model. Therefore, the correlation should be improved if we use a higher quality model for the variation in mistuning.

#### 4.3 Ten Disk Model

Next we formed a much better statistical model of the mistuning by using FMM ID to measure the blade frequency deviation in all 10 test disks. Again, the data was found to be normally distributed. In this case, the sample standard deviation was 1.92%, which is much closer to the true standard deviation of 2%. Since this sample standard deviation is based off of 10 times as much data as the crude model, our uncertainty in the parameter has been greatly reduced. The 95% confidence interval ranges from 1.77% to 2.12%.

The Monte Carlo simulations were then repeated with this improved statistical model. The resulting CDFs are shown in Fig. 11. As expected, we see substantial improvement in the correlation of the 1E and 3E simulations with experimental data. Furthermore, simulated CDFs from the 6E and 9E cases are virtually identical to those generated from a much cruder statistical model. This result confirms that the 6 and 9E cases are insensitive to statistical modeling errors. Again, the simulations agree well with the experimental benchmark. Therefore, the FMM based probabilistic analysis process may be used to accurately determine the statistical behavior of the fleet.

#### 4.4 Sensitivity Dependence on Mistuning Level

We found throughout Section 4 that the accuracy of a probabilistic mistuning analysis depends on two factors: the quality of the statistical model, and sensitivity of the system's response to statistical modeling errors. As shown in Fig. 10, the system's sensitivity is a function of the engine order of excitation as well as the probability range of interest. However, it should be noted that the sensitivity regime is also governed by the level of mistuning in the system.

Consider the CDFs shown in Fig. 12. Each curve corresponds to the response of a system with a different mistuning standard deviation,  $\sigma_1$  and  $\sigma_2$  respectively. In the case of  $\sigma_1$ , the 99<sup>th</sup> percentile amplitude is about 1.5. Yet, the 99<sup>th</sup> percentile amplitude for  $\sigma_2$  is slightly higher. Therefore, we can plot the 99<sup>th</sup> percentile of the response as a function of the standard deviation of the mistuning, Fig 13. Figure 13 shows the 99<sup>th</sup> percentile for all four engine orders considered in this study. Notice that in the vicinity of 2% mistuning, the 6E and 9E curves have a near-zero slope. Thus, the 99<sup>th</sup> percentile amplitude in these cases is insensitive to small changes in the mistuning level (standard deviation). This is consistent with the sensitivity plot shown in Fig. 10. However, if the mistuning was instead on the order of 0.5%, then Fig. 13 indicates that the response to a 6E or 9E excitation would be much more sensitive to changes in the standard deviation. Thus, a system's sensitivity to errors in the statistical modeling depends on the level of mistuning.

### 5. CONCLUSIONS

It was shown that FMM and FMM ID may be used for probabilistic analysis of mistuned bladed disks. The process involves using FMM ID to collect data on the mistuning and tuned frequencies of a population of bladed disks. This data is then used to construct statistical models of the parameters. Finally, we can use those

statistical models with FMM to perform Monte Carlo simulations of the fleet response. FMM is an ideal physical model for Monte Carlo simulations because it is accurate, simple to use, and extremely efficient.

The method was verified experimentally by comparing the results of our Monte Carlo simulations against laboratory measurements of mistuned disks. The FMM approach worked very well. We found that the accuracy of the method depends on both the quality of the statistical model, and the sensitivity of the system's response to errors in the statistical modeling. The sensitivity regime may be assessed through the sensitivity analyses discussed in Sections 4.2 and 4.4. The efficiency of FMM makes these analyses fast and easy to perform. If it is found that the system is sensitive, then the statistical models may need to be improved to ensure an accurate simulation. Such improvements can be made by using FMM ID to measure the mistuning of additional hardware. Conversely, additional testing may not be necessary on systems that are insensitive to modeling errors.

FMM and FMM ID were experimentally shown to be effective tools for probabilistic analysis of mistuned bladed disks.

## REFERENCES

- [1] Srinivasan, A. V., 1997, "Flutter and Resonant Vibration Characteristics of Engine Blades," *Journal of Engineering for Gas Turbines and Power*, **119**, 4, pp. 742-775.
- [2] Yang, M.-T., and Griffin, J. H., 2001, "A Reduced Order Model of Mistuning Using a Subset of Nominal Modes," *Journal of Engineering for Gas Turbines and Power*, **123**(4), pp. 893-900.
- [3] Feiner, D.M., and Griffin, J. H., 2002, "A Fundamental Model of Mistuning for a Single Family of Modes," *Journal of Turbomachinery*, **124**(4), pp. 597-605.
- [4] Castanier, M. P., Ottarsson, G., and Pierre, C., 1997, "A Reduced Order Modeling Technique for Mistuned Bladed Disks," *Journal of Vibration and Acoustics*, **119**(3), pp. 439-447.
- [5] Petrov, E., Sanliturk, K., Ewins, D., and Elliott, R., 2000, "Quantitative Prediction of the Effects of Mistuning Arrangement on Resonant Response of a Practical Turbine Bladed Disk," 5th National Turbine Engine High Cycle Fatigue Conference, Chandler, Arizona.
- [6] Seinturier, E., Lombard, J.P., Berthillier, M., and Sgarzi, O., 2002, "Turbine Mistuned Forced Response Prediction: Comparison With Experimental Results," ASME Paper 2002-GT-30424, International Gas Turbine Institute Turbo Expo, Amsterdam, The Netherlands.
- [7] Judge, J.A., Pierre, C., and Ceccio, S.L., 2002, "Mistuning Identification in Bladed Disks," *Proceedings of the International Conference on Structural Dynamics Modeling*, Madeira Island, Portugal.
- [8] Kim, N.E., Griffin, J.H., 2003, "System ID in High Modal Density Regions of Bladed Disks," 8<sup>th</sup> National Turbine Engine High Cycle Fatigue Conference, Monterey, California.
- [9] Feiner, D.M., and Griffin, J.H., 2003, "Mistuning Identification of Bladed Disks Using a Fundamental Mistuning Model – Part I: Theory," *Journal of Turbomachinery*, **126**(1).
- [10] Feiner, D.M., and Griffin, J.H., 2003, "Mistuning Identification of Bladed Disks Using a Fundamental Mistuning Model – Part II: Application," *Journal of Turbomachinery*, **126**(1).
- [11] Jones, K.W., and Cross, C.J., 2003, "Traveling Wave Excitation System for Bladed Disks," *Journal of Propulsion and Power*, **19**(1), pp. 135-141.



# FIGURES

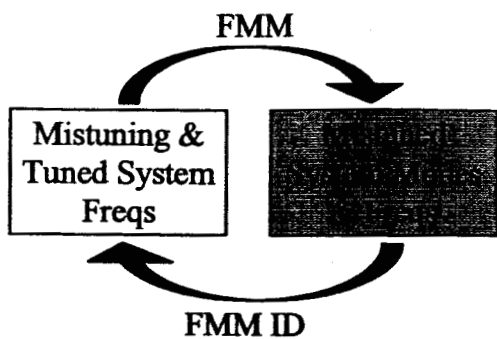


Figure 1: Schematic representation of the relation between FMM and FMM ID.

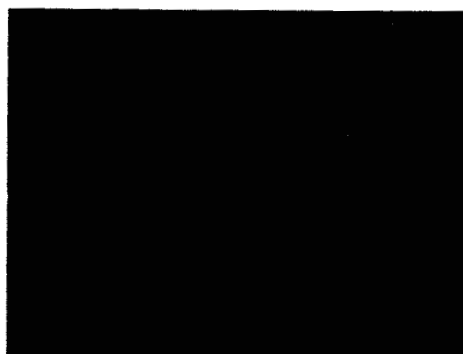
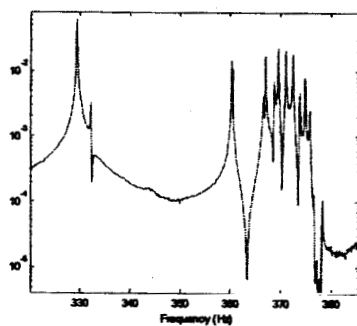
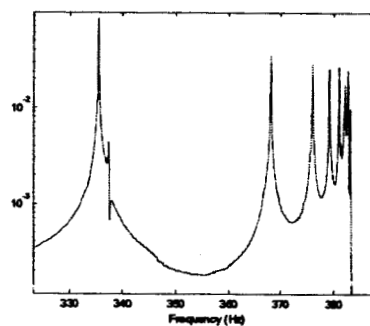


Figure 2: Test IBR.



(a) Before Tuning



(b) After Tuning

Figure 3: FRF's of the test IBR before and after tuning.

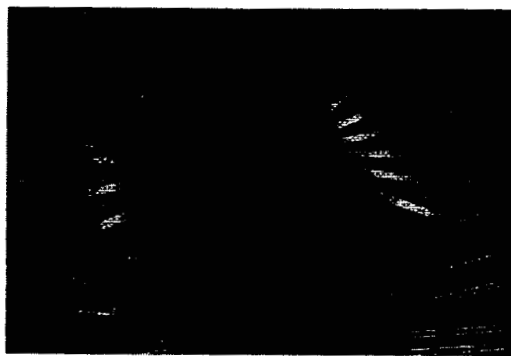
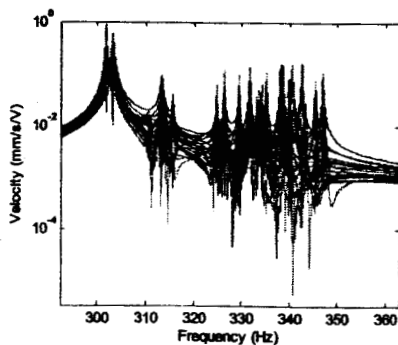
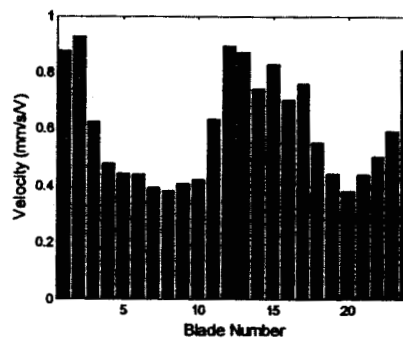


Figure 4: Test IBR surrounded by an array of excitation magnets.

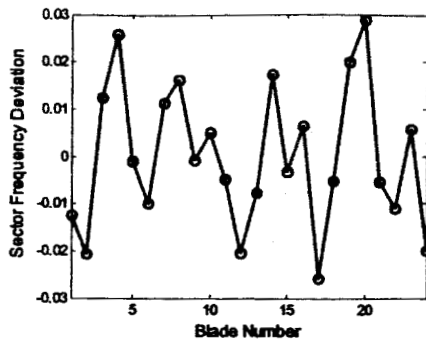


(a) FRFs of all blades

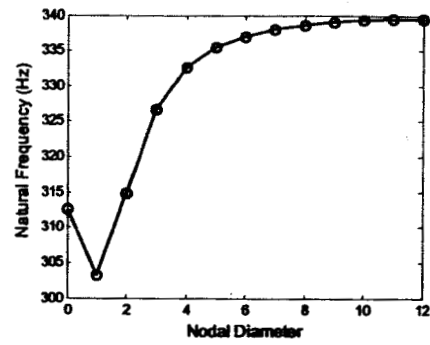


(b) Peak amplitude of each blade

Figure 5: Representative measurements of one disk configuration, driven by a 1E excitation.



(a) Mistuning



(b) Tuned system frequencies

Figure 6: Parameters of test disk as determined through FMM ID

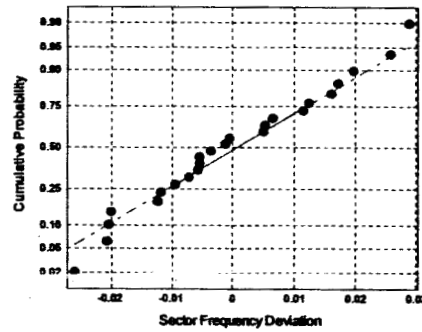


Figure 7: Normal plot of blade frequency deviations of one disk

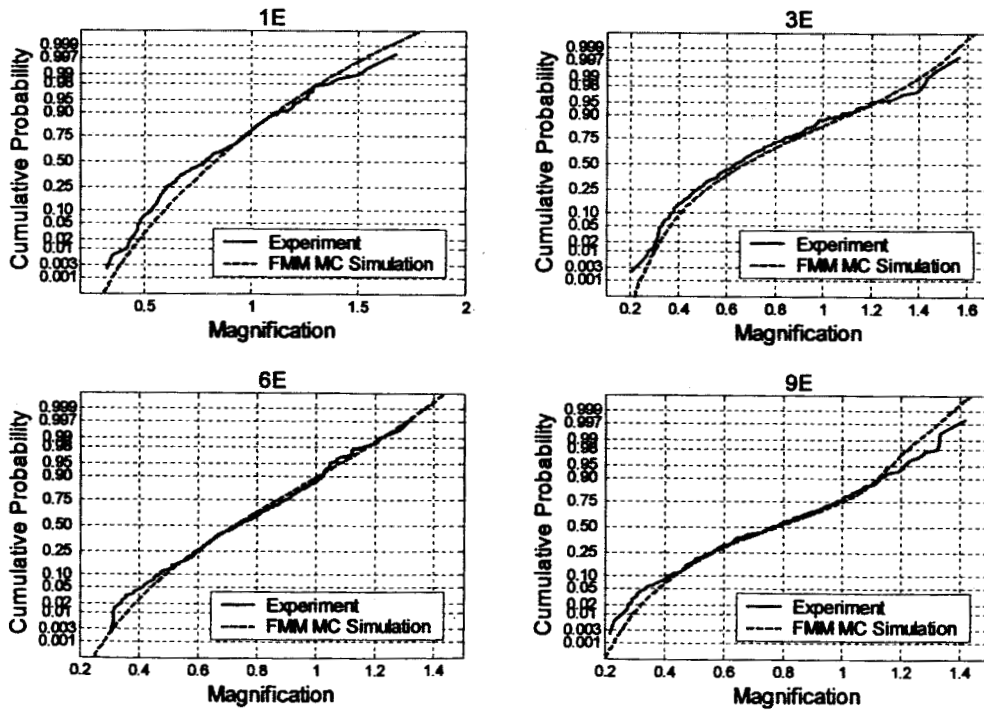


Figure 8: Comparison of the experimental and simulated CDFs of the peak blade amplitudes

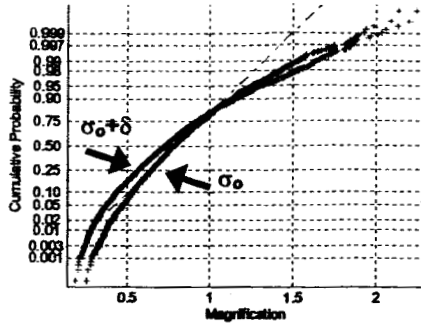


Figure 9: Change in CDF due to perturbation in standard deviation

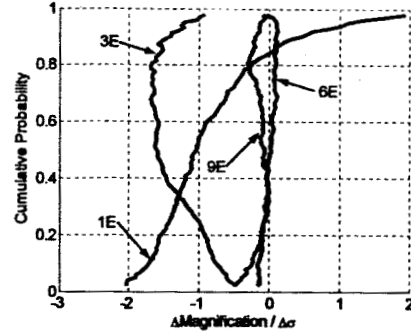


Figure 10: Sensitivity of CDF to perturbation in standard deviation, centered about  $\sigma=2\%$

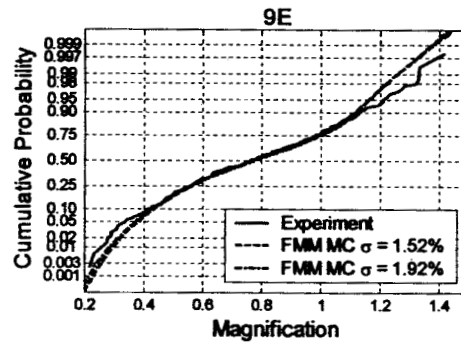
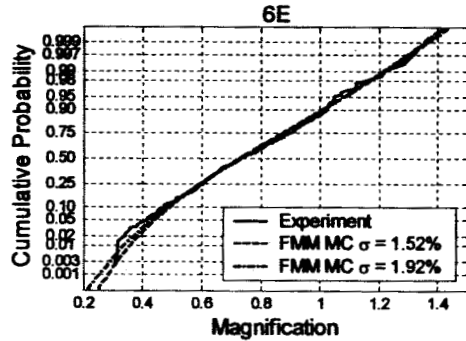
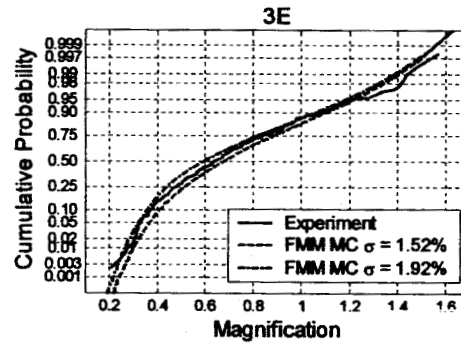
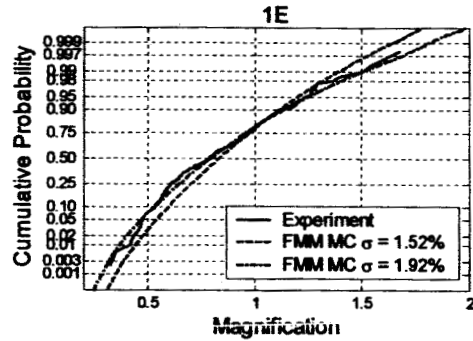


Figure 11: Comparison of the experimental and simulated CDFs of the peak blade amplitudes based on both the crude and improved statistical models.

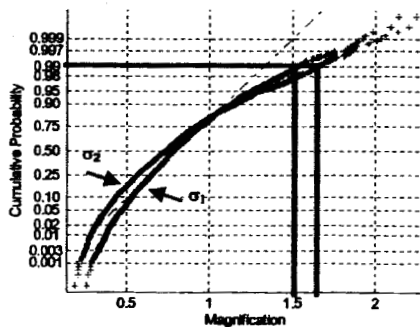


Figure 12: Change in 99<sup>th</sup> percentile amplitude due to change in mistuning standard deviation

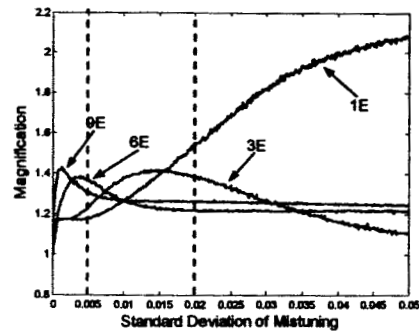


Figure 13: 99<sup>th</sup> percentile amplitude as a function of mistuning standard deviation